SOLUTION OF THE PROBLEM OF TRANSIENT HEAT TRANSFER BETWEEN A SOLID BODY AND A SURROUNDING STREAM

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A method of electrical simulation is shown by which one can solve the conjunctive problem of transient heat transfer between a solid body and a surrounding stream of fluid.

In the analysis of a laminar stream around a body there arises the problem of determining the temperature field of both the body and the fluid. Such a problem reduces to a simultaneous solution of the equation of heat convection in the fluid and of heat conduction in the solid. The steady-state solution to such so-called conjunctive problems has been considered in [1].

Analytical methods of solving a transient conjunctive problem have been developed with certain simplifications, assuming, for example, a uniform heating of the channel wall [2] or a small ratio of body thickness to fluid layer thickness [3]. Whenever such assumptions may lead to large errors, it becomes necessary to resort to computer-aided solutions, but these are very time consuming.

In this article we will show how the problem of transient heat transfer between a body of arbitrary shape and a surrounding fluid stream can be solved by an electrical simulation. We propose to use for this purpose existing models and, specifically, a USM-1 analog computer [4] with some structural modifications.

We consider a laminar flow through a thick-walled tube of rectangular cross section, the walls of the tube being heated by the fluid. In the approximation of the boundary-layer theory we have the following equation [5]:

$$\frac{\partial \Theta}{\partial \tau} + v(y) = a_F \frac{\partial^2 \Theta}{\partial y^2} \tag{1}$$

$$(0 \leqslant \tau \leqslant \infty,\ 0 \leqslant y \leqslant h,\ 0 \leqslant x \leqslant d).$$

The temperature of the oncoming fluid can be expressed as

$$\Theta|_{\tau=0} = \Theta_0;\ \Theta|_{x=0} = \Theta_0. \tag{2}$$

The equation of heat conduction in the solid body is

$$\frac{\partial T}{\partial \tau} = a_S \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \tag{3}$$

$$(0 \leqslant \tau < \infty;\ -l \leqslant y \leqslant 0;\ 0 \leqslant x \leqslant d).$$

The boundary conditions for the body are

$$T|_{\tau=0} = T_0; \tag{4}$$

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = \left|\left.\frac{\partial T}{\partial x}\right|_{x=d} = \left.\frac{\partial T}{\partial y}\right|_{y=-l} = 0; \tag{5}$$

$$\Theta|_{y=+0} = T|_{y=-0}; \tag{6}$$

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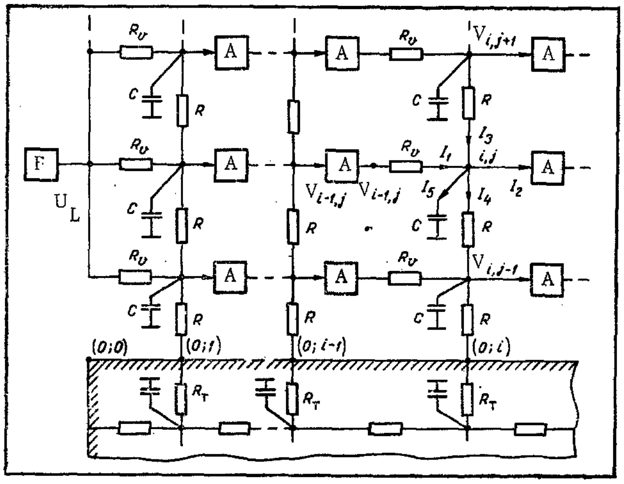

Fig. 1. Schematic block diagram of a model for analyzing the heat transfer between a solid body and a laminar stream of fluid.

$$-\lambda_F \frac{\partial \Theta}{\partial y}\bigg|_{y=+0} = -\lambda_S \frac{\partial T}{\partial y}\bigg|_{y=-0}. \tag{7}$$

Inside the rectangle $[0 \le x \le d,\ 0 \le y \le h]$ we draw two families of straight lines $x = i\Delta x$ and $y = j\Delta y$ ($i = 0, 1, 2, \ldots, n$ and $j = 0, 1, 2, \ldots, m$), where $n\Delta x = d$ and $m\Delta y = h$. The intersection point of line $x = i\Delta x$ with line $y = j\Delta y$ will be called a network node with coordinates (i, j). Replacing the derivatives in Eq. (1) by finite-difference ratios at the network nodes, we obtain

$$\frac{\partial \Theta_{i,j}}{\partial \tau} + \frac{v(j\Delta y)}{\Delta x}\left(\Theta_{i,j} - \Theta_{i-1,j}\right) = \frac{a_F}{\Delta y^2}\left(\Theta_{i,j+1} + \Theta_{i,j-1} - 2\Theta_{i,j}\right). \tag{8}$$

Let us examine the model shown schematically in Fig. 1. Here the tube wall is simulated by a plain RC-network for solving Eq. (3) [6]. The node points with coordinates i, j = 0 represent the solid–fluid interface. At these points is coupled on another network of passive RC-sections and a unity-gain amplifier A. For the currents into each node of this network we can write the following Kirchhoff equation:

$$I_1 + I_3 = I_2 + I_4 + I_5. \tag{9}$$

Since

$$I_1 = \frac{V_{i-1,j} - V_{i,j}}{R_v};\quad I_3 = \frac{V_{i,j+1} - V_{i,j}}{R};\quad I_4 = \frac{V_{i,j} - V_{i,j-1}}{R};\quad I_5 = C\frac{\partial V_{i,j}}{\partial t},$$

and current I_2 is negligible on account of the rather high amplifier input impedance, hence Eq. (9) can be transformed into

$$\frac{\partial U_{i,j}}{\partial t} + \frac{1}{R_v C}\left(V_{i,j} - V_{i-1,j}\right) = \frac{1}{RC}\left(V_{i,j+1} + V_{i,j-1} - 2V_{i,j}\right). \tag{10}$$

A comparison of (8) and (10) shows that, when conditions

$$\frac{v(j\Delta y)}{\Delta x} = \frac{n}{R_v C} \text{ and } \frac{a_f}{\Delta y^2} = \frac{n}{RC} \tag{11}$$

are satisfied, the part of the network in Fig. 1 above the boundary points simulates Eqs. (8) describing the thermal processes in the fluid. Voltage V_L at the output of the shaping circuit F is an electrical analog of the temperature in the oncoming stream and may, generally, vary in time.

If the parameters of the RC-network which simulates the solid body are matched so as to satisfy the relation

$$\frac{R_S}{R} = \frac{\lambda_F \Delta y}{\lambda_S \Delta y_S},$$

following the condition of thermal flux balance at the interphase boundary (7), then the electrical model in Fig. 1 can be regarded as a model for the simultaneous solution of Eqs. (1) and (3) with the boundary

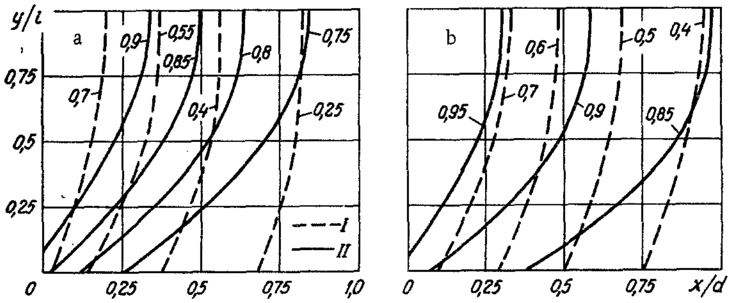

Fig. 2. Isotherms for the flow of fluid through a flat channel: a) τ = 1 h 20 min; b) τ = 2 h; I) v = 0.5 m/h; II) 1.5 m/h.

conditions (4)-(7), i.e., for the solution of the transient conjunctive problem. We note that one can satisfy condition (6) by directly connecting the upper and the lower network through their terminal resistances at points (1, 0), (2, 0), . . . , (i, 0).

This model was used for analyzing the heat transfer between water flowing through a symmetrical flat channel (2h = 0.04 m) and a solid body (d = 0.6 m, l = 1.1 m) with the following thermophysical properties: ρ = 7900 kg/m^3, c = 0.13 J/kg · °C, λ = 45 W/m · °C. The flow velocity was assumed equal to the mean-discharge velocity at all points.

The model was designed around a USM-1 analog computer. Its distinct feature is the RC-network for simulating the fluid stream and including a unity-gain amplifier. The function of this network is to set the boundary conditions at the solid–fluid interface.

Although no provision is made in the basic compter for setting the boundary conditions in this manner, this component can be added to the USM-1 device without difficulty. Thus, each unity-gain amplifier is made up here of an amplifier–summator from a function generator unit (FG) in series with a channel for generating boundary conditions of the first kind (GB-1). Such a hookup yields the proper gain and greatly simplifies the switchboard layout. Furthermore, with the amplifier (FG and GB-1) in a series connection it is not necessary to invert the input signal – as is required in the conventional operation of the model.

In Fig. 2 are shown results of the proposed simulation procedure. A few isotherms for the fluid stream are shown here (the temperatures are indicated in relative units) for T_0 = 0.

The proposed method makes it possible to greatly extend the use of existing models and to adapt them for solving conjunctive transient heat transfer problems.

NOTATION

Θ is the temperature of the fluid;
τ is the real time, h;
v is the velocity of fluid;
h is the channel half-width;
d is the channel length;
l is the wall thickness;
V_{ij} is the voltage at the network node with coordinates i, j;
t is the machine time;
T is the temperature of the solid body;
n is the time scale factor;
λ is the thermal conductivity;
c is the specific heat;
ρ is the density;
a is the thermal diffusivity.

Subscripts

F denotes fluid;
S denotes solid;

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